Holographic dark energy: quantum correlations against thermodynamical description

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Abstract

Classical and quantum entropic properties of holographic dark energy (HDE) are considered in view of the fact that its entropy is far more restrictive than the entropy of a black hole of the same size. In cosmological settings (in which HDE is promoted to a plausible candidate for being the dark energy of the universe), HDE should be viewed as a combined state composed of the event horizon and the stuff inside the horizon. By any interaction of the subsystems, the horizon and the interior become entangled, raising thereby a possibility that their quantum correlations be responsible for the almost purity of the combined state. Under this circumstances, the entanglement entropy is almost the same for both subsystems, being also of the same order as the thermal (coarse grained) entropy of the interior or the horizon. In the context of thermodynamics, however, only additive coarse grained entropies matter, so we use these entropies to test the generalized second law (GSL) of gravitational thermodynamics in this framework. While we find that the original Li's model passes the GSL test for a special choice of parameters, in a saturated model with the choice for the IR cutoff in the form of the Hubble parameter, the GSL always breaks down.

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The concept of holographic principle, first formulated by 't Hooft [1] and Susskind [2] as a possible window to quantum gravity, has become part of the mainstream after the Malcadena's discovery of AdS/CFT duality [3]. In attempt to reconcile it with the success of effective-quantum-field-theory description of elementary-particle phenomena, the holographic principle becomes a quantitative measure of the overabundance of degrees of freedom in ordinary quantum field theory (QFT). Since black holes appear to involve a vast number of states that are not describable within ordinary QFT, the entropy for an effective QFT $\sim L^3\Lambda^3$, where L is the size of the region (providing an IR cutoff) and Λ is the UV cutoff, should obey the upper bound [4]

$$L^3\Lambda^3 \leq L^{3/2}M_{Pl}^{3/2} \sim (S_{BH})^{3/4} \ll S_{BH} \ , \tag{1}$$

where S_{BH} is the entropy of a black hole of the size L. Since the entropy in QFT scales extensively, it is clear that in an expanding universe Λ should be promoted to a varying quantity (some function of L to manifest the UV/IR connection), in order (1) not to be violated during the course of the expansion. This gives a constraint on the maximum energy density in the effective theory, $\rho_\Lambda \sim \Lambda^4$, to be $\rho_\Lambda \leq L^{-2}M_{Pl}^2$. Obviously, ρ_Λ is the energy density corresponding to a zero-point energy and the cutoff Λ. Such a framework gave rise to a variable cosmological-constant (CC) approach generically dubbed 'holographic dark energy' (HDE) [5, 6], which has proved since to have a potential to shed light both on the 'old' CC problem [7] and the 'cosmic coincidence problem' (CCP) [8].

The main reason of why the above HDE model is so appealing in possible description of dark energy is when the bound (1) is saturated ρ_Λ gives the right amount of dark energy in the universe at present, provided $L \simeq H^{-1}$, where H is the Hubble parameter. Moreover, since ρ_Λ is a running quantity, it also has a potential to substantially alleviate the CCP. On the other hand, the most problematic aspect of the saturated HDE model is its compatibility with a transition from decelerated to accelerated expansion. Indeed, as it is well known, the identification of the IR cutoff with the Hubble parameter for spatially flat universes (as suggested by observations) leads to unsatisfactory cosmologies. In this case one is not able to explain either the accelerating expansion of the present universe for non-interacting fluids [5] or a fact that the acceleration era has set in just recently, for interacting fluids. A more realistic class of models, which do allow transitions between the cosmological eras, is provided by the non-saturated HDE scenario [9, 10]. As a way out of the above problems, a

suggestion of setting L at the future event horizon has been widely accepted [11], although inconsistency with matter dominance irrespective of the choice for L was claimed in [10] for any saturated model.

In the present paper, we consider a question of smallness of the upper bound (1) (with respect to S_{BH}) from the aspect of information theory [12]. Using the formalism and language of the physics of information we define fine/coarse grained entropies as well as the entropy of entanglement for HDE. Finally, we switch to classical (thermodynamical) description of the system to test the generalized second law (GSL) of gravitation and irreversibility for the HDE scenario.

The central question we would like to address here is why the entropy (1) is so much smaller than the entropy calculated using the first law of thermodynamics with the temperature of the horizon (the only temperature we have at our disposal). The latter turns out to be of the order of S_{BH} as well (see below). Note that the original model [4] leading to (1), aiming to explain the present acceleration of the universe (to become HDE), leads to cosmological models which do have finite event horizons. Therefore in cosmological settings, in which the system described by (1) becomes HDE, we actually deal with two subsystems: the horizon and the stuff inside the horizon [1]. We will argue that quantum mechanical entanglement between the two subsystems can explain the small value in (1).

Let us now analyze the situation from the aspect of information theory. The small entropy in (1) is usually referred as a fine grained entropy of the composite system, and since it is $\ll S_{BH}$ (as well as $\ll$ than other entropies to be defined below), we will assume, for simplicity, that the composite system is in a pure state. The results from information theory [12] then easily apply to our case. The subsystems (the interior and the horizon), are not generally described by pure states but by mixed density matrices, resulting in an entanglement entropy (or fine grained entropy) for the subsystems, -$Tr\rho_{\Lambda}log\rho_{\Lambda}$ and -$Tr\rho_{hor}log\rho_{hor}$. It measures both the degree of entanglement between the subsystems and the departure from a pure state for a particular subsystem. Furthermore, if the initial state of the combined system is pure, the equality of the entanglement entropies results, $-Tr\rho_{\Lambda}log\rho_{\Lambda} = -Tr\rho_{hor}log\rho_{hor} \equiv S^{ent}$ [12]. In addition, S^{ent} can be also thought of as a lack of information I, defined as

[1] We shall deal here only with the CC stuff inside the horizon since during dark-energy domination its contribution grossly overwhelms that of ordinary matter.

$I = S^{ther} - S^{ent}$, where S^{ther} is the thermal (or coarse grained) entropy, representing a distribution which maximizes the entropy for a given system at a given average energy. Thus, $S^{ther} > S^{ent}$. Note that any thermodynamic considerations involve only S^{ther}'s. The purity of the combined state and the presence of entanglement may result that a great deal of information, $2S^{ent}$, to be stored in the correlations between the subsystems rather than in the subsystems themselves. Therefore if the information content of the correlations equals $2S^{ent}$, the correlations between subsystems would make the whole system pure.

Next, let us estimate S^{ent} for the case under consideration. Although it is not unambiguously defined because of a lack of knowledge of the system, the information theory says [12] that $S^{ent} \simeq S_{BH}$ or S^{ther}_{Λ}, depending on the share the subsystems have in the whole system (see also [13, 14]). S^{ther}_{Λ} can be obtained using the first law of thermodynamics with the temperature of the horizon, giving a contribution of the order of S_{BH} (see below) [2]. Thus, S^{ent} is typically of the same order as the horizon entropy.

After having shown qualitatively that quantum correlations between the event horizon and the interior dark energy given by a HDE variable Λ term, may be responsible for a small value (1), we turn to a quantitative analysis involving classical (thermodynamical) properties of HDE. Namely, we put the HDE model under the scrutiny of another profound physical principle, the GSL of gravitational thermodynamics. In the context of modern cosmology, the Second Law of thermodynamics is manifest there since the initial conditions for cosmology have low entropy, so we can see the Second Law in operation [15]. In the problem under consideration it is adequate to invoke the GSL because we are dealing with cosmologies in which ever accelerating universes always possess future event horizons. The GSL states that the entropy of the event horizon plus the entropy of all the stuff in the volume inside the horizon cannot decrease in time. The idea of associating entropy with the horizon area surrounding black holes is now extended to include all event horizons [16].

We aim to restrict the parameter c^2, which helps to parametrize the saturated HDE energy density $\rho_{\Lambda} = (3/8\pi)c^2 L^{-2} M^2_{Pl}$ [6], by assuming the validity of the GSL. A restriction on c^2 under the combined phenomenological constraints has been recently obtained [17] for certain HDE models. For another studies searching for the conditions required for validity of the GSL in cosmological models involving dark energy, see [18].

[2] For possible ambiguities see the footnote on p.5. Thus, either S^{ther}_{Λ} or $|S^{ther}_{\Lambda}|$ is of the order of S_{BH}.

As mentioned earlier, the GSL involves only thermal entropies which are additive by definition, and with a macroscopic scale of resolution due to coarse graining, the arrow of increasing time and irreversibility naturally emerge. The GSL thus states that (omitting 'ther' from S_Λ^{ther} hereafter)

$$\dot{S}_{hor} + \dot{S}_\Lambda \geq 0 \; . \tag{2}$$

Here overdots represent time derivatives, $S_{hor} = \pi M_{Pl}^2 d_E^2$ and the future event horizon is given by

$$d_E = a \int_a^\infty \frac{da}{a^2 H} \; , \tag{3}$$

with a being a scale factor.

The entropy inside the horizon can be determined using the first law of thermodynamics

$$T_\Lambda dS_\Lambda = d(\rho_\Lambda V) + p_\Lambda dV \; , \tag{4}$$

where T_Λ is the horizon temperature, $V = (4\pi/3)d_E^3$ and $p_\Lambda = w_\Lambda \rho_\Lambda$. We shall examine (4) using both the event and the apparent horizon in the definition of the temperature $T_\Lambda \equiv 1/(2\pi d_{E,A})$, where $d_A = H^{-1}$ for flat space. Putting all together, the constraint (2) can be written as [3]

$$d_{E,A}\left(-d_E^2 L^{-3}\dot{L} + \frac{3}{2}(1 + w_\Lambda)L^{-2} d_E \dot{d}_E\right) + \frac{1}{c^2}\dot{d}_E \geq 0 \; . \tag{5}$$

For the choice $L = d_E$ and using $d_E = c(1+r)^{1/2} d_A$, obtained from the Friedmann equation (for flat space) with the dominant matter component ρ_m and $r = \rho_m/\rho_\Lambda$, the constraint (5) is reduced further to

$$(d_{A,E}/d_E)(1 + 3w_\Lambda) + 2/c^2 \geq 0 \qquad ; \qquad \dot{d}_E > 0 \; , \tag{6}$$

$$(d_{A,E}/d_E)(1 + 3w_\Lambda) + 2/c^2 \leq 0 \qquad ; \qquad \dot{d}_E < 0 \; . \tag{7}$$

[3] Actually for the time derivative of S_Λ, we obtain from (4) that $\dot{S}_\Lambda = (1/2T_\Lambda)c^2 \dot{L}(1 + 3w_\Lambda)M_{Pl}^2$, showing that for $\dot{L} > 0$ S_Λ starts decreasing at the onset of the accelerated phase ($w_\Lambda \leq -1/3$). Taking $T_\Lambda \sim L^{-1}$ as usual, we find upon integration (neglecting an integration constant) that $S_\Lambda \sim c^2 L^2 (1 + 3w_\Lambda) M_{Pl}^2$, which is obviously negative. To our knowledge, a situation where S_Λ is negative even in non-phantom cosmologies was indicated for the first time in [19]. In this case, the thermal entropy, which should obviously reflect the number of microscopically distinct quantum states, becomes hard to interpret. This also has implications for information theory introduced above.

Let us now test the popular Li's model [6], with $r = 0$, $w_\Lambda = -1/3 - 2/3c$, $d_E \sim a^{1-1/c}$, against the GSL. One obtains,

$$-c(d_{A,E}/d_E) + 1 \geq 0 \quad ; \quad c > 1 \, , \tag{8}$$

$$-c(d_{A,E}/d_E) + 1 \leq 0 \quad ; \quad c < 1 \, . \tag{9}$$

Taking first $T_\Lambda = 1/(2\pi d_E)$, we do obtain a contradiction for both constraints (8) and (9) unless $c = 1$; the model therefore passes the GSL test only for $c^2 = 1$. With $T_\Lambda = 1/(2\pi d_A)$ one obtains zero on the LHS of either constraint (8-9). This means that total thermodynamical entropy of the system stays constant during cosmological evolution in the Λ-dominated phase. The GSL is therefore respected for any c^2.

Another plausible choice, $L = H^{-1}$, makes sense only in the presence of interaction between (near) pressureless dark matter with HDE [9, 20]. Otherwise HDE is not able to bring about an accelerated phase of the present universe [5]. To obtain a realistic cosmology, a certain degree of non-saturation in the HDE energy density is also needed, to result in a matter-dominated epoch in the past [9, 10]. Since we are going to test the model under GSL only during accelerated expansion, we shall use the saturated version of HDE. In this case, the constraint (2) reduces to

$$d_{E,A}\left(d_E^2 H\dot{H} + \frac{3}{2}(1+w_\Lambda)H^2 d_E \dot{d}_E\right) + \frac{1}{c^2}\dot{d}_E \geq 0 \, . \tag{10}$$

For a constant interaction parameter, it follows that $\rho_m, \rho_\Lambda \propto a^{-3m}$ with $m = 1 + c^2 w_\Lambda$ [9]. Also $m < 2/3$, to obtain an accelerated universe. Using this, all the relevant entries in (10) can easily be obtained. Taking $T_\Lambda = 1/(2\pi d_A)$, (10) is reduced further to

$$3c^2 - 1 \geq 2c^2 \, , \tag{11}$$

leading to a final constraint $c^2 \geq 1$. However, with the aid of the Friedmann equation for flat space, one can express c^2 for such a choice for L as

$$c^2 = \frac{1}{1 + r_0} \simeq 0.7 \, . \tag{12}$$

Hence, the GSL is not respected here.

Another choice in (10), $T_\Lambda = 1/(2\pi d_E)$, leads to a bound

$$9w_\Lambda^2 c^4 + (2 + 12w_\Lambda)c^2 + 1 \geq 0 \, , \tag{13}$$

which now depends on w_Λ. Observationally, w_Λ is very close to -1, and $w_\Lambda = -1$ is also the most natural value for HDE (since in the original derivation it represents zero-point energies). This means that c^2 should reside in the allowable range, $1 < c^2 < 1/9$. Since the value (12) obtained from the Friedmann equation does not fit the above range, we see again that the GSL is not respected. So, the saturated HDE model with the choice for the IR cutoff, $L = H^{-1}$, does not respect the GSL of gravitational thermodynamics.

Let us conclude by laying stress once again on some basic points on which this paper resides. We have shown that the entropy for the HDE model as given by (1) should not be used in thermodynamical considerations. Instead, it should be interpreted as the fine grained entropy of the system composed of the horizon and the interior dominated by a variable CC term. Stated differently, even if we assume thermal equilibrium between weakly interacting subsystems, the whole system will not be thermal. We have also introduced the entanglement entropy for the subsystems (their fine grained entropy) to show that, via quantum correlations, this entropy may be responsible for the (almost) purity of the entire system. The fine grained entropies for the subsystems are neither additive nor conserved. On the other hand, any thermodynamical consideration does involve only thermal (or coarse grained) entropies, which are additive but, of course, not conserved. Using these properties we have tested the model against the GSL, which requires that the thermal entropy of the whole system (the sum of thermal entropies of the subsystems in thermal equilibrium) never decreases in the course of cosmic expansion. We have tested two simplest although distinct models (non-interacting versus interacting), to obtain that the model in which the IR cutoff is set by the future event horizon, always has a capacity to pass the GSL test.

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[1] G. 't Hooft gr-qc/9311026.

[2] L. Susskind, J. Math. Phys. (NY) 36 (1995) 6377.

[3] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.

[4] A. Cohen, D. Kaplan, and A. Nelson, Phys. Rev. Lett. 82 (1999) 4971.

[5] S. D. Hsu, Phys. Lett. B594 (2004) 1.

[6] M. Li, Phys. Lett. B603 (2004) 1.

[7] See, e.g., S. Nobbenhuis, gr-qc/0609011 and references therein.

[8] P. J. Steinhardt, in 'Critical Problems in Physics", edited by V. L. Fitch and Dr. R. Marlow (Princeton University Press), Princeton, N. J., (1997).

[9] D. Pavón and W. Zimdahl, Phys. Lett. B628 (2005) 206.

[10] B. Guberina, R. Horvat and H. Nikolić, JCAP 0701 (2007) 012.

[11] For perfect fluids see, [6]; Q. G. Huang and M. Li, JCAP 0408 (2004) 013; for interacting fluids see e.g., B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B624 (2005) 141; H. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B632 (2006) 605; H. M. Sadjadi and M. Honardoost, gr-qc/0609076.

[12] See, e.g., W. H. Zurek: "Information transfer in Quantum Measurements: Irreversibility and Amplification" in *Quantum Optics, Experimental Gravitation and Measurement Theory*, Eds: P. Meystre and M. O. Scully (Plenum Press, New York, 1983); M. A. Nielsen and I. L. Chuang: "Quantum Computation and Quantum Information" (Cambridge University Press, Cambridge, 2000); F. Alexander and J. D. Farmer, physics.gen-ph/0708.2837.

[13] J-W. Lee, J. Lee and H-C. Kim, JCAP 0708 (2007) 005.

[14] S. Mukohyama, M. Serin and H. Kodama, Phys. Rev. D55 (1997) 7666.

[15] See e.g., T. Banks, hep-th/0701146 and references therein.

[16] See, e.g., P. C. W. Davies, Class. Quant. Grav. 4 (1987) L225; *ibid.* 5 (1988) 1349.

[17] Y-Z. Ma and Y. Gong, astro-ph/0711.1641; X. Zhang and F-Q. Wu, Phys. Rev. D72 (2005) 043524; Phys. Rev. D76 (2007) 023502; Z. Chang, F-Q. Wu and X. Zhang, Phys. Lett. B633 (2006) 14.

[18] R. Brustein, Phys. Rev. Lett. 84 (2000) 2072; T. M. Davies, P. C. W. Davies and C. H. Lineweaver, Class. Quant. Grav. 20 (2003) 2753; G. Izquierdo and D. Pavón, Phys. Lett. B639 (2006) 420; B. Guberina, R. Horvat and H. Nikolić, Phys. Lett. B636 (2006) 80; M. R. Setare, Phys. Lett. B641 (2006) 130; JCAP 0701 (2007) 023; H. M. Sadjadi, Phys. Lett. B645 (2007) 108; Y. Gong, B. Wang and A. Wang, Phys. Rev. D75 (2007) 123516; R. Horvat, Phys. Lett. B648 (2007) 374; H. M. Sadjadi, Phys. Rev. D76 (2007) 104024; J. A. de Freitas Pacheco and J. E. Horvath, gr-qc/0709.1240; S. He and H. Zhang, hep-th/0708.3670.

[19] Y. Gong, B. Wang and A. Wang, JCAP 0701 (2007) 024.

[20] R. Horvat, Phys. Rev. D70 (2004) 087301.